

Rashba Spin Interferometer

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Abstract—A spin interferometer utilizing the Rashba effect is proposed. The novel design is composed of a one-dimensional (1D) straight wire and a 1D half-ring. By calculating the norm of the superposed wave function, we derive analytical expressions to describe the spin interference spectrum as a function of the Rashba coupling strength. Presented spin interference results are identified to include (i) the quantum-mechanical 4π rotation effect, (ii) geometric effect, and (iii) Shubnikov-de Haas-like beating effect.

I. INTRODUCTION

The Rashba spin-orbit coupling effect [1], originating from the effective magnetic field generated by the electric field under the relativistic transformation [2], [3], has stimulated plenty of works in semiconductor spintronics [4]. For its experimentally proved gate-voltage tunability [5], rotating the spins by Rashba effect (the Rashba spin precession [6]) or even controlling the spin direction via the gate voltage [7] are theoretically possible. It turns out that countless interesting spin phenomena based on the Rashba effect have been proposed, including the spin interference effect.

Nitta *et al.* first raised the issue of spin interferometer considering a quantum ring with strong Rashba effect [8]. Later the analysis was extended from an ideal one-dimensional (1D) ring to a two-dimensional ring under first quantization [9], and second quantization [10]. Another design of the spin interferometer using the square loop was also proposed [11], and is even experimentally proved recently [12].

In this paper, we propose another design for the Rashba spin interferometer, which can be analytically solved. With obtained formulae, we identify the spin interference into three categories: (i) quantum-mechanical 4π rotation effect, which states that a quantum state ket needs a rotation of 4π to bring it back, instead of 2π [3], [13]; (ii) geometric effect, which arises from the geometry-dependent phase difference; (iii) Shubnikov-de Haas (SdH)-like beating effect, which is the most distinctive feature of our proposal.

II. THEORETICAL CALCULATION

Consider an interferometer device composed of a quantum wire attached by a quantum half-ring, where the Rashba spin-orbit coupling is present. See Fig. 1(a). An ideal spin injector is set up at one of the two connection points of the ring-wire structure, while spin interference is expected to occur at the other one. Assume that the injected spin, described by a state ket $|s\rangle^i$, propagates through both the wire path, yielding the state ket $|s\rangle_x^w$ at position x , and the ring path, yielding $|s\rangle_\theta^r$ at the angle θ , with equal probability. Applying Ref. [14], i.e., spatially translate the injected spin which is assumed to be

projected to the states at the Fermi level [15], we can write down the individual state kets. For the straight wire, we have

$$|s\rangle_x^w = e^{i\bar{k}_w x} \sum_{\sigma} e^{-i\frac{\sigma\Delta\theta_w}{2}} \langle\sigma; \phi=0|s\rangle^i |\sigma; \phi=0\rangle \quad (1)$$

with $\bar{k}_w \equiv (k_+^w + k_-^w)/2$ and $\Delta\theta_w \equiv 2m^*\alpha_w x/\hbar^2$, where α_w is the Rashba strength within the wire, and k_{\pm}^w are the spin-dependent wave vectors. In the present work, we will assume constant Rashba strength in both the wire and the ring to facilitate the analytical derivation. Spatial dependence of the Rashba coupling, if considered, can be straightforwardly handled by the contour-integral method [14]. For the half-ring, we similarly have

$$|s\rangle_\theta^r = e^{i\bar{k}_r \pi\theta} \sum_{\sigma} e^{-i\frac{\sigma\Delta\theta_r}{2}} \langle\psi_\sigma; \phi=\pi/2|s\rangle^i |\psi_\sigma; \phi=\pi/2-\theta\rangle \quad (2)$$

with $\bar{k}_r \equiv (k_+^r + k_-^r)/2$ and $\Delta\theta_r \equiv 2m^*\alpha_w \pi\theta/\hbar^2$, where we denote the Rashba coupling strength within the ring as α_r , which in general may differ from α_w . In both Eqs. (1) and (2), the well-known Rashba eigenspinor is given by $|\psi_\sigma; \phi\rangle \doteq (-ie^{i\phi}, \sigma)^\dagger / \sqrt{2}$, where ϕ is the direction angle of the electron wave vector. (Along x axis we set $\phi=0$.)

Using Eqs. (1) and (2), one can depict the spin vectors along individually the ring and the wire paths and clearly see the purpose of our design of combining one straight wire and one half-ring. In Fig. 1(b), we inject an x -polarized spin, which propagates precessionlessly along the half-ring and precessingly along the wire. Conversely, if we inject a y -polarized spin, the precessing and precessionless situations switch, as shown in Fig. 1(c). When tuning the Rashba coupling strength (via the gate voltage [5]), which is equivalent to an effective magnetic field, the spin coming out from the wire (ring) is rotated while that from the ring (wire) is fixed

Fig. 1. (a) Schematic sketch of the spin interferometer device. Individual spin vectors calculated on the half-ring and the wire are plotted in (b) and (c) with an injected x -polarized and y -polarized spins, respectively.

in the case of x -polarized (y -polarized) injection of spin. We therefore expect to see the effect of the quantum-mechanical rotation of the state ket with 4π periods [13], mapped from the external magnetic field in vacuum to the effective magnetic field in solids.

We proceed by deriving the explicit form of $|SI\rangle$. At the interference region, the spin-interfered state is superposed by $|SI\rangle = |s\rangle_{x=2R}^w + |s\rangle_{\theta=\pi}^w$, regardless of normalization. Next we define the dimensionless parameter

$$\delta_{w(r)} \equiv 4m^* \alpha_{w(r)} R / \hbar^2, \quad (3)$$

which are responsible for the Rashba coupling and characterize the precession angle of the injected spin $\Delta\theta_w = \delta_w$ and $\Delta\theta_r = \pi \delta_r / 2$. Furthermore, under the assumption that the spin is injected at E_F so that we have $\bar{k}_{w(r)} = \sqrt{2m^* / \hbar^2} \sqrt{E_F + \hbar^2 \delta_{w(r)}^2 / (32m^* R^2)}$, we can define

$$\Delta_{w(r)} \equiv \bar{k}_{w(r)} R = \frac{\delta_{w(r)}}{4} \sqrt{1 + E / \delta_{w(r)}^2}, \quad (4)$$

where $E \equiv 32m^* R^2 E_F / \hbar^2$ is dimensionless. Therefore, we have the spin-interfered state

$$\begin{aligned} |SI\rangle &= e^{i2\Delta_w} \sum_{\sigma} e^{-i\frac{\sigma\delta_w}{2}} |\psi_{\sigma}; 0|s\rangle^i |\psi_{\sigma}; 0\rangle \\ &+ e^{i\pi\Delta_r} \sum_{\sigma} e^{-i\frac{\sigma\pi\delta_r}{4}} |\psi_{\sigma}; \pi/2|s\rangle^i |\psi_{\sigma}; -\pi/2\rangle \end{aligned} \quad (5)$$

We now consider the specific cases of spin injection, namely, the x -polarized and the y -polarized injection of spin. Substituting $|s\rangle^i \doteq \begin{pmatrix} 1 \\ 1 \end{pmatrix} / \sqrt{2}$ into Eq. (5), one can, after some mathematical manipulation, obtain the spin-interfered state ket $|SI; x\rangle$, yielding the spin interference

$$\langle SI; x | SI; x \rangle = 2 - 2 \sin(\delta_w / 2) \cos(\pi \Delta_r - 2\Delta_w - \pi \delta_r / 4). \quad (6)$$

Similarly, with $|s\rangle^i = \begin{pmatrix} -i \\ 1 \end{pmatrix} / \sqrt{2}$ we have

$$\langle SI; y | SI; y \rangle = 2 - 2 \sin(\pi \delta_r / 4) \cos(\pi \Delta_r - 2\Delta_w - \delta_w / 2). \quad (7)$$

III. RESULTS AND DISCUSSION

In the following we will use Eqs. (6) and (7), together with Eqs. (3) and (4), to demonstrate the 4π rotation effect, the geometric effect, and the SdH-like beating effect in the proposed interferometer device. Specifically, we consider InGaAs-based materials with the electron effective mass $m^* / m_e = 0.03$ (m_e is the electron rest mass) and the maximal Rashba coupling strength 0.03 eV nm [16].

A. 4π Rotation Effect: Separate Control

Consider the interferometer with the Rashba strength of the wire (or δ_w) tunable, while that of the ring (or δ_r) fixed. The ring radius is set $R = 1 \text{ } \mu\text{m}$. We inject the spin at $E_F = 5 \text{ eV}$, and tune α_w from 0 to 0.03 eV nm , corresponding to about $\delta_w = 0 \rightarrow 15\pi$. Considering the x -polarized injection of spin, the 4π rotation effect is clearly seen, as shown in Fig. 2(a). Figure 3(a) also shows the 4π rotation effect in the case of y -polarized injection of spin. In general, the period of

Fig. 2. Spin interference spectrum as a function of δ_w by injecting an x -polarized spin. In the separate control cases (a) and (c), δ_r is fixed as the maximum of δ_w while in the on-board tuning cases (b) and (d), δ_r is set equal to δ_w . Labels “high energy” and “low energy” correspond to $E_F = 5 \text{ eV}$ and $E_F = 32 \text{ meV}$, respectively.

4π exists whenever we tune the Rashba strength only on one side. Referring to Eq. (5) and noting that $\Delta_{w(r)} \approx \sqrt{E} / 4 = \text{constant}$ for the given E_F here [see Eq. (4)], one can see that the 4π period stems from the partial wave $|s\rangle_{2R}^w$ passing through the wire. When superposing $|s\rangle_{2R}^w$ with an arbitrary fixed state (not parallel to $|s\rangle_{2R}^w$), the oscillation with the 4π period, which can never be obtained by such a normalized state $|s\rangle_{2R}^w$ only, is revealed.

B. Geometric Effect: On-Board Tuning

We now tune δ_r and δ_w simultaneously ($\delta_r = \delta_w$) via an on-board gate voltage, with other parameters identical with the previous discussion. In this case the oscillation behavior is completely changed, as can be seen in Figs. 2(b) for the x -polarized case and 3(b) for the y -polarized case. Such an interference effect originates from the difference in $\Delta\theta_w$ and $\Delta\theta_r$ shown in Eqs. (1) and (2), and is therefore a geometric effect. Note that here since $\Delta\theta_r - \Delta\theta_w = (\pi/4 - 1/2) \delta_w =$

Fig. 3. Same as Fig. 2, except injecting a y -polarized spin.

$(\pi - 2)\delta_w/2$, which is an irrational number of multiple of δ_w , the state can no longer be returned to its original state, in principle.

Note the distinct interference outcomes between Figs. 2(a) and 2(b), subject to exactly the same spin vectors along the individual wire and ring paths [see Figs. 1(b)]! In this x -polarized case, whether tuning δ_r or not, the spin vectors along the ring are always fixed, since one of the spin channel is blocked. Specifically, the expansion coefficients for the ring path are $\langle \psi_\sigma; \pi/2 | s \rangle^i = \delta_{1,\sigma}$ (the Kronecker delta symbol defined by $\delta_{m,n} = 0$ for $m \neq n$ and $\delta_{m,m} = 1$) so that $|s\rangle_\pi^r = e^{i\pi(\Delta_r - \delta_r/4)} \begin{pmatrix} -1 \\ 1 \end{pmatrix} / \sqrt{2}$ carries the information of *fixed* spin directions, no matter the phase is tuned or not. However, superposing $|s\rangle_{2R}^w$ with $|s\rangle_\pi^r$ will eventually bring the interference spectrum into $\langle SI | SI \rangle = 2 + 2 \operatorname{Re} \left(\frac{w}{2R} \langle s | s \rangle_\pi^r \right)$, which depends on whether $|s\rangle_\pi^r$ is varying or not.

C. Shubnikov-de Haas-Like Beating Effect

Finally, we arrive at another feature of our spin interferometer, namely, the beating effect. If we extend the range of δ_w by considering a larger R , say $R = 5 \mu\text{m}$, and inject the spin at a lower energy, say $E_F = 32 \text{ meV}$, one can clearly see the beating effect in the x -polarized case using the separate control, as shown in Fig. 2(c). On the contrary, such a beating effect cannot be observed when injecting a y -polarized spin, whether using the separate control or the on-board tuning.

Mathematically, the beating comes from the modulation of $\cos(\pi \Delta_r - 2\Delta_w - \pi \delta_r/4)$ on $\sin(\delta_w/2)$ in Eq. (6) for $\langle SI; x | SI; x \rangle$, while Eq. (7) manifests that $\langle SI; y | SI; y \rangle$ does not exhibit any beating since $\cos(\pi \Delta_r - 2\Delta_w - \delta_w/2)$ is oscillating with δ_w but $\sin(\pi \delta_r/4)$ is simply constant. Note also that such a beating vanishes when injecting a high energy spin such that $\Delta_w \approx \text{constant}$ as in the previous cases, since the cosine part in Eq. (6) becomes a constant. Physically, the beating effect is similar to that in the SdH oscillation. In the ordinary SdH effect, longitudinal resistance ρ_{xx} oscillates with the increasing applied perpendicular magnetic field B_\perp , which turns the step-function-like density of states into (broadened) Landau levels. With either spin-orbit coupling or an in-plane magnetic field B_\parallel , each Landau level splits into two peaks due to spin, and the oscillation of ρ_{xx} will compose of two close but different frequencies, leading to the beating.

In the present Rashba spin interferometer, increasing δ_w by the gate voltage is to increase the effective magnetic field, and we can thus map B_\perp and ρ_{xx} in the ordinary SdH onto the Rashba field strength (or δ_w) and $\langle SI | SI \rangle$ in our SdH-like beating effect, respectively. In the latter case, the phase factors of the two spin components in the Rashba-tuned wave function (the state $|s\rangle_{2R}^w$ here) are of frequencies proportional to their momentum $\hbar(\vec{k}_w \pm \Delta k)$ with Δk being the wave vector difference proportional to α_w . When superposing $|s\rangle_{2R}^w$ with the fixed $|s\rangle_\pi^r$ state, the two closely spaced frequencies lead to the beating, if the two spin components are equally occupied. This is why we cannot observe the beating in the y -polarized case. (One spin channel along the wire is blocked so that only one frequency exists.)

IV. CONCLUSION

In conclusion, we have shown the spin interference due to Rashba effect in our proposed device. Obtained interference spectrum includes the 4π rotation, the geometric, and the SdH-like beating effects. Presented results suggest experimental measurement of simply the electric current through the device versus the Rashba coupling strength, using either separate control or on-board tuning. The former shows 4π periods while the latter does not, when injecting higher energy spins with any polarization. SdH-like beating is expected only when injecting x -polarized (actually, also valid for z -polarized) spins at low energy. The only nontrivial requirement is to tune δ by an enough wide range, which may require a larger ring, a stronger spin-orbit coupling, or even a heavier effective mass. To maintain the system as ballistic, meaning that the ring radius is of the order of micro-meter, estimation of δ from Eq. (3) shows that InAs- or InGaAs-based materials seem to be the most feasible candidate. GaAs-based material is less suggested, and SiGe/Si/SiGe symmetric quantum wells are impossible in reality.

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